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# STATUS REPORT ON CONTROL OPTIMIZATION, STABILIZATION AND COMPUTER ALGORITHMS FOR AIRCRAFT APPLICATIONS

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## SUMMARY

This status report contains a brief and informal description of the research carried out by faculty, staff, and students of the MIT Laboratory for Information and Decision Systems under NASA Grant NGL-22-009-124. The period covered in this status report is from April 15, 1981 to May 15, 1982. The research suport is provided by the NASA Ames and Langley Research Centers.

The research objectives are to advance the state of the art in the analysis and design of complex multivariable reliable control systems and is directly motivated by the need for high-performance and fault-tolerant aircraft systems. In addition to the description of the theoretical research, this status report outlines a preliminary feasibility study of the design of a lateral control system for a VTOL aircraft that is to land on a DD963 class destroyer under high sea-state conditions.

In the main body of the report we summarize recent progress in the following areas.

1. VTOL Control System Design Studies
2. Progress in Robust Multivariable Control System Synthesis
3. Adaptive Control Systems
4. Failure Detection Algorithms
5. Fault-Tolerant Optimal Control Theory.

# 1. LATERAL CONTROL SYSTEM DESIGN FOR VTOL LANDING ON A DD963 IN HIGH SEA STATES

## 1.1 Motivation

The landing of VTOL aircraft on small platforms (e.g., the DD963 class destroyer) represents an extremely challenging multivariable control problem, especially if the landing is to take place in the presence of high sea states (e.g., sea state 5) and in the presence of high winds. Needless to say, extreme reliability is required.

This problem of VTOL landing was selected for detailed analysis under the auspices of this grant 3 years ago. The objective of the study was to use the VTOL landing problem as a specific case study in which the theoretical aspects of our research could be evaluated, and further insight into relevant theoretical research on robustness and reliability could be obtained. We do not intend to produce an "engineering design".

The aircraft is the Lift/Cruise Fan shaft coupled Research Technology Aircraft for which adequate nonlinear dynamic models are available and were provided to MIT by NASA Ames Research Center.

The long range goal of this research effort is to completely specify what we mean by a reliable landing control system, which must take into account aircraft/ship sensor interfaces, and control strategies. Thus many of the theoretical results outlined in other sections of this report pertaining to robust control, adaptive control, failure detection, fault-tolerant control are important for the VTOL landing problem.

The first phase of this research effort was completed in 1980 and is fully documented in the S.M. thesis of McMuldroch. Two landing concepts ("chase-the-deck" and "landing on a peak") were investigated using only longitudinal dynamics and under the assumptions that all aircraft and ship state variables are available for measurement in a centralized controller. The analysis did include the effects of ground effects which were not found to be very significant.

Our studies indicated that key quantities, such as thrust-to-weight ratio and control system bandwidth, are very sensitive to ship dynamics. Better stochastic dynamic models for ship motion were needed to carry out realistic control and estimation studies. Towards this goal, since September 1979 Prof. Triantayllou (Ocean Engineering Department, MIT) has joined the research team to provide better models for ship dynamics under different environmental conditions, and initial modeling efforts have been initiated.

In the remainder of this section we summarize our progress in the lateral dynamics area [1], [2], [3], [4], carried out by Professors Athans and Triantafyllou and Mr. Bodson.

## 1.2 Lateral Ship Dynamics

The lateral dynamics of the ship, i.e., the roll, sway, yaw motions have been approximated by state space models. These motions are within linear theory decoupled from the vertical motions (heave and pitch) and the surge motion which is of higher order.

The coupling among roll, sway and yaw is very strong and the coupling coefficients are frequency dependent. Of particular importance is the roll motion which due to the slenderness of the vessel can achieve large values (20 or 30 degree amplitude). In such cases the linear prediction is poor primarily because the linear damping is very small, while the quadratic fluid drag becomes predominant. The response in roll is then dominated by the nonlinear damping (overdamped system) so that the method of harmonic balance has been used successfully to predict the nonlinear roll response. The technique of harmonic balance is attractive because it is an easy extension of the linear theory.

It is very important to model the lateral dynamics in their coupled form because once the equivalent damping coefficient for roll has been evaluated, the coupling coefficients between roll and sway, and roll and yaw modify accordingly the sway and yaw response.

The approximation of the linear coefficients presented the same characteristics described in the case of the heave-pitch motions: They are frequency dependent, resulting in higher order models once a rational approximation has been achieved.

The exciting forces were also frequency dependent. In addition, although the vessel responds at the frequency of encounter, the amplitude of the forces depends on the wave frequency, thus introducing additional complexity in the system. This important feature was found to be a source of major discrepancies if not modelled appropriately.

The sway, roll and yaw forces and moments depend on the wave slope, i.e., the square of the wave frequency, while sway and yaw lack

hydrostatic restoring forces. As a result, a direct modeling resulted in zero-pole cancellations and all related problems. By proper adjustment of the modelling procedure a minimum order model was obtained.

For the sea the same model as in the case of heave-pitch was used, i.e. a sixth order state space model closely fitting the Bretschneider spectrum once the modal frequency and significant wave height have been specified.

The overall model of lateral motions, by introducing appropriate simplification, was reduced to a 16 order state space model with inputs the ship speed and direction relative to the waves, and the Bretschneider spectrum characteristics.

The equivalent roll damping coefficient was determined using data from other existing vessels.

A computer program was created and a Kalman filter was designed for some typical sea conditions, to reconstruct the state from noisy measurements of the sway, roll, yaw motions. The performance of the filter depends primarily on an accurate estimate of the modal frequency of the wave spectrum.

## 1.3 Motion Prediction

The models developed for heave-pitch and sway-roll-yaw can be used to predict a few seconds ahead the future dynamics of the vessels.

The predictability of the motion depends primarily on the bandwidth of their transfer functions, thus making roll the most predictable motion.

If the state is known prefectly, then by plotting the error covariance of the predicted motions versus prediction time it was found that a reasonably good prediction can be achieved for all motion 5 seconds ahead, except for roll which can be predicted 10 seconds ahead.

In the case of a few noisy measurements, the prediction ability is degraded. By using only three measurements, roll, sway, and yaw, with significant noise, it was found that the prediction time goes down to about 2 seconds for all motions except for roll for which the prediction time is about 8 seconds.

For heave and pitch it was found that with two noisy measurements the prediction time was also about 2-3 seconds.

This can be viewed as a lower limit and in a practical application we can achieve typically a 3 second prediction time for all motions except roll for which it can go to 8 seconds.

The strong dependence on the modal frequency of the spectrum suggests the need for a good estimator of the modal frequency.

The parameters of the spectrum change slowly so that a different time scale Kalman filter can be used to estimate those parameters. Significant questions to be answered include the performance of the filter in the case of directional seas, given that the present model assumes unidirectional waves; the performance in the case of multiple peaks such as in the case of swell. It should be noted that the effect of a directional sea on the estimation of the ship motions was found to be rather small, so that an

alternative scheme using an extended Kalman filter to estimate the sea parameters could be advantageous.

Final documentation of all results pertaining to ship dynamics, estimation, and prediction will take place during the summer of 1982.

## 1.4 VTOL Lateral Control System [1]

The landing of small VTOL aircraft on destroyers is an extremely challenging problem if it is to be realized under high sea state conditions and zero visibility. Without special aids, this task is almost impossible for a human pilot.

There are basically two possible strategies in the solution of this problem. The first is to leave to the pilot the complete control of the aircraft, but help him with advanced displays. These give him information about the aircraft position and attitude, as well as those of the ship (and possibly some prediction of the ship motions). They may also indicate some desirable flight path (flight director). Advanced controls may be provided to partly relieve the pilot from the high load of controlling the VTOL aircraft. NASA Ames is doing research in this area.

The second strategy is to leave the task of landing the aircraft completely to an automatic controller. The role of the pilot is then to supervise the correct landing of the aircraft. This would allow him to take care of other tasks he might not have been able to carry out otherwise.

Note that both strategies could be mixed. For example, the tracking of the lateral ship motions may be left to an automatic controller/tracker, while the task of vertical tracking and landing would be left to the pilot, possibly with the help of some display indicating him the present and future ship vertical position.

In Bodson's thesis [1], supervised by Prof. Athans, the emphasis is focused on the design of an automatic controller. A previous study has addressed the problem of the longitudinal motions, i.e. the motions in the vertical plane. The most significant ship motion in this plane is the vertical motion, called heave. The pitch motion is not very significant, except for the heave motion it induces at the landing pad (which is significantly behind the ship center of rotation). The present study addresses the ship motion tracking problem for the lateral case. Then, the most significant motion is the ship roll motion, which can be very large. The lateral translation motion, called sway, is also important, especially due to the large sway component induced by the roll at the landing pad (located above the ship center of rotation).

The challenge of the tracking of the ship motions by the VTOL lies in the strong limitations of the control authority available, in the high level of the perturbations (wind disturbances, ground effects, ship airwake), in the strong couplings present in the system, and in the need for a highly robust control system.

Usually, studies of this problem use loop-by-loop control system designs, using classical control theories in which the controller ignores the internal couplings of the system. Similarly, the issue of robustness is often addressed on a loop-by-loop basis, but almost never in a real multivariable sense (although individual loop stability margins may not represent at all the overall system stability margins). The design process used in our approach does not suffer from such limitations.

The limitations on the available control authority justify the use of some optimization criterion, and of related modern control theories (LQ/LQG). These methods have the advantage of naturally handling multivariable systems, and of recognizing the coupling present in such systems. Some recent results in the analysis of the robustness of multivariable systems (and its improvement for LQG based designs) are also important tools in the design of control systems operating under critical conditions.

The purpose of Bodson's thesis was not to produce an engineering design. Nor does it provide new theoretical results. It illustrates how modern control theories and related recent results can be used to design a truly multivariable control system for such an advanced application, and evaluate the controller performance and robustness. This work also analyzes the physical constraints of the tracking process of the lateral ship motions. These constraints are independent of the control system design methodology adopted. The requirements and physical limitations related to the VTOL landing problem are studied.

Although Bodson's thesis mainly details the design of an automatic controller, the accuracy achievable in the prediction of ship motions is also assessed, as it is a key element in any piloted VTOL landing.

The main contributions of the research are:

- the derivation of an accurate ship model that retains the stochastic nature of the ship motions, and the couplings amongst them

- the analysis of an optimal predictor of the ship motions for applications in piloted landings, and the assessment of lower bounds on the prediction errors

- the design of an optimal controller/tracker for applications in automatic landings, the indication of the tradeoffs betwwen tracking errors and control authority, and the analysis of the important couplings and physical constraints related to the tracking of the lateral ship motions

- the demonstration of the use of the singular values analysis, and the robustification procedure, to obtain a robust control system.

In general, it is easy to design a robust multivariable control system that forces the VTOL to track with very small error the yaw and the sway motion of the landing pad; we remark that the sway motion is the most significant. However, there is a basic problem in forcing the VTOL roll motion to match that of the landing pad. This, we feel, is an important problem which to the best of our knowledge has not been discussed in the literature, and we briefly indicate the nature of the problem below (for more details see Ref. [1]).

The basic problem is that the greatest contribution of the sway motion of the landing pad is induced by the ship's roll motion, because the landing pad is significantly above the ship's center of rotation. On the other hand, the proper deflection of the VTOL thrust vector to compensate for sway error, induces a roll motion to the VTOL which happens to be about 180° out of phase with the roll motion of the landing pad. Thus, simultaneous good sway motion and roll angle tracking do not appear to be physically possible.

The impact of roll tracking errors has to be studied further with respect to the its effect on the landing gear (the VTOL may land on one gear) and, of course, one must be careful not to have the VTOL wing touch the deck.

The above problems deserve further study. It is important to stress that such limitations are not unique to the automatic landing system; rather, they are a consequence of the ship dynamics and of the VTOL dynamics. Many simulations do not use precise ship dynamics; rather the motion of the ship is simulated via the combination of sinusoids of different frequency. This would not lead to the exact relationship between roll-induced sway for the landing pad, and misleading conclusions could be drawn.

## 2. MULTIVARIABLE CONTROL SYSTEMS

### 2.1 Background

The design of multivariable control systems, i.e. systems with several inputs and several outputs, is of fundamental importance in aircraft and space applications. The VTOL problem that we discussed in Section 1 is one of many applications that can benefit from a systematic design methodology for multivariable control systems. Other applications include problems of

(a) Integrated flight control for aircraft
(b) Helicopter control

which are of interest to NASA Langley and Ames research staff.

In the past year several important results have been obtained in the area of multivariable control which represent fundamental contributions to properties of multivariable control designs that deal with performance improvement and robustness to modeling errors. Our progress is highlighted in the remainder of this section.

### 2.2 Progress in Multivariable Root Loci [5] to [9]

Professors Stein, Sastry, Levy and their students have developed several useful analytical properties of the root locus method for multivariable feedback systems. Of course, the root-locus method for single-input-single-output (SISO) systems has traditionally been a very helpful tool for both analysis and design of servomechanisms.

Although the root locus concept generalizes to the multiple-input-multiple-output (MIMO) case, the resultant multivariable root-loci are far more complex than their SISO counterparts. The basic fundamental reason is that MIMO root loci are defined over several "leaves" (Rieman surfaces) of the s-plane, and thereby their interpretation is much more complex. Our research, described in more detail in Refs. [5] to [9] has developed much needed insight is this important area.

## 2.3 Progress in Multivariable Robustness: [10] to [13]

Many fundamental new results dealing with robustness for MIMO systems have been obtained by Professors Athans and Stein and several of their graduate students.

The importance of obtaining robustly stable feedback control systems has long been recognized by designers. Indeed, a principal reason for using feedback rather than open-loop control is the presence of model uncertainties. Any model is at best an approximation of reality, and the relatively low order, linear, time-invariant models most often used for controller synthesis are bound to be rather crude approximations.

More specifically, a given system model can usually be characterized as follows. There is a certain range of inputs typically bounded in amplitude and in a certain frequency range for which the model is a reasonable engineering approximation to the system. Outside of this

range, due to neglected nonlinearities and dynamic effects, the model and system may behave in grossly different ways. Unfortunately, this range of permissible inputs is rarely spelled out explicitly along with the model, but is rather implicit in the technology that the model came from - there is no "truth in modelling" low in systems theory.

The term robustness as used in this section will refer to the extent to which a model of a open-loop system may be changed from the nominal design model without destabilizing the overall closed-loop feedback system designed to control the outputs of the open-loop system. We stress that in this definition, we implicitly assume that the dynamic compensator is fixed, that is, it does not change if, for whatever reason, one suspects that the actual open-loop dynamics are different from those used in the model. Real time changes in the compensator structure (gains or other changes) lead to adaptive control systems, a topic that will be addressed in Section 3 of this progress report. Thus, the term robustness refers to the preservation of closed-loop system stability in the face of model uncertainty not accounted for in the compensator design.

Robustness issues are not new in control system design. In classical single-input, single-output (SISO) servomechanism designs, robustness specifications were often specified in terms of gain margin and phase margin requirements. However, for multiple-input, multiple-output (MIMO) control systems, similar robustness measures are not straight forward, and their interpretation must be done with care.

The robustness problem [11] can be logically divided into three distinct questions:

(a) given a model of a feedback control system how close to instability is it?

(b) given the class of model errors for which the control system is stable, does this class include the model errors that can be reasonably expected for this particular system?

(c) how can a robust feedback system be designed?

Question (a) is an analysis problem that can be solved exactly by an appropriate mathematical formulation. Question (b) cannot be answered without a proper understanding of the physics of the physical system to be controlled and the assumptions that were made in constructing a model to be used in controller design. Even with a good understanding of modelling deficiencies it is difficult to characterize this knowledge in a form that is mathematically easy to deal with from the analysis point of view. Question (c) combines aspects of both questions (a) and (b) in that a designer must be able to tell if there exists a controller that would be able to tolerate the class of modelling errors he believes is reasonable for a given open-loop system design model.

However, the robustness properties of a feedback system cannot be optimized without regard to the deterministic and noise performance requirements for the control system. For open-loop stable systems, this is clearly demonstrated since the most robust control system is the

open-loop system with no feedback. Of course, for this open-loop stable system the transient response to a step input command or the response to disturbances may not meet the performance specifications. This underscores the fact that there is a fundamental tradeoff between robustness, deterministic performance and stochastic performance (performance with respect to stochastic disturbance and/or sensor noise inputs). Specification of any one of these system characteristics may place constraints on the achievable performance or margin of stability for the other two system characteristics. For example, with linear-quadratic-gaussian (LQG) regulators one may obtain acceptable deterministic responses to command inputs and have an adequate margin of stability but the adequate robustness properties may be obtained at the expense of an increased response to process noise driving the open-loop plant if the deterministic performance must be maintained.

In signal-input, single-output (SISO) control system design these issues are well understood. The classical frequency domain techniques for SISO design naturally handle the robustness characterization. These techniques employ various graphical means (e.g., Bode, Nyquist, inverse-Nyquist, Nichols, diagrams) of displaying the system model in terms of its frequency response. From these plots, it is very easy to determine (by inspection) the minimum change in model frequency response that leads to instability. From the same plots the system's transient response and response to various inputs can also be estimated. Thus,

the classical control system designer can observe the fundamental tradeoffs that must be made from these plots.

This is in contrast to the multiple-input, multiple-output (MIMO) case where these tradeoffs are often obscured. Many design techniques for MIMO system such as pole placement completely neglect the robustness issue in placing poles to obtain a good transient response. Other state space methods attempt to overcome this problem by using state-space models whose parameters may vary and then assuring that for a range of parameter values the closed-loop feedback system will be stable. However, these parameterized state-space models cannot characterize modelling errors arising from neglected dynamics and, therefore, omit an important class of variations in the nominal design model for stability analysis. In short, many state space methods do not naturally lead to techniques that adequately account for modelling error.

The presently available frequency domain MIMO design techniques also have the problem that they do not ensure stability for a sufficiently large class of modelling errors. They basically treat a MIMO system as a series of single-loop design problems that are essentially decoupled. They give good stability margins in a coordinate system that makes the design problem simple but <u>not</u> in the coordinate system of the input and output of the physical plant, <u>the coordinate system in which it is important to have robustness and good stability margins</u>. For this reason, these methods may not detect small modelling errors that could potentially destabilize the closed-loop feedback system. The measures of the robustness

of a MIMO feedback control system that we have developed [11],[12] do not suffer from the above deficiency; they will always detect the near instability of a feedback control system. However, since in many cases these robustness tests are conservative, a significant amount of our research was devoted to eliminating this conservatism. These results are derived in the frequency domain using a multivariable version of Nyquist's criterion, singular values and the singular value decomposition familiar from numerical linear algebra.

The main contributions of our research are:

(1) a simplified derivation of available and new robustness results for linear time-invariant systems.

(2) the unification of these robustness results under a common framework based on a classification of various types of modelling errors

(3) the reduction of conservatism of robustness results using only information about the magnitude of modelling error by including information about the structure of the modelling error

(4) the interpretation of robustness properties of LQG control systems via the framework based on model error type.

The result obtained summarize and extend the state of the art on the robustness of multivariable control system. However, the practical application of these results is far from trivial and requires sound engineering judgment about the nature of modelling errors based on the

<u>physics of the controlled system</u>. However, it is hoped that practical experience with physical systems may provide further insight as to how to successfully apply these new results since engineering knowledge about modelling errors is not always easily interpreted in the mathematical framework required by these results.

The guaranteed robustness properties of Linear Quadratic (LQ) regulators and Kalman filters (KF) were extended [13] to a different formulation which involves an exponential time-weighting of the quadratic penalty in LQ control problems, and (dually) an exponential variation upon the process and measurement noise intensity matrix in filtering problems. These designs have the useful property that all closed-loop poles of either the LQ regulator or Kalman filter can be guaranteed to be at a certain distance from the imaginary ($j\omega$) axis of the s-plane. The multivariable root loci and robustness properties of such designs with a guaranteed degree of stability have been derived in Ng's S.M. thesis [13].

# 3. ADAPTIVE CONTROL SYSTEMS

## 3.1 Background

The development of a systematic design methodology for the synthesis of practical self-adjusting control systems which can maintain first stability and second performance improvement. In the presence of rapid and large variations in the open-loop dynamics, adaptation represents a very important generic goal in control systems engineering, in view of its wide applicability to industrial and defense applications. The so-called "adaptive control problem" has received attention by theoreticians and practitioners alike for the past 25 years. About a dozen books and hundreds of articles have been devoted to the subject; different philosophies have been developed (model reference adaptive control, self-tuning regulators, dual-control methods, multiple-model adaptive control etc.) and a variety of (mostly academic) examples have been simulated.

In spite of the intense research activity, it is the opinion of the authors (who have actively contributed to the literature) that there is a significant gap between the available methodologies and the potential applications. To put it bluntly, we do not believe that any of the available adaptive control algorithms can be routinely implemented on a real system and guarantee even the stability of the closed loop process in the presence of the inevitable unmodeled high frequency dynamics.

One should not blame the theory for this state of affairs. Elegant and useful theoretical advances have been made in the last decade, and

especially in the past three years, that have unified diverse approaches. The difficulty appears to be that some of the hypotheses needed to rigorously prove the theoretical results are too restrictive from a practical point of view. Hence, new advances in the theory are necessary, by making different assumptions with better reflect the desired properties of physical control systems.

By practical we mean that the adaptive control loop must adjust its bandwidth (crossover frequency) in such a manner so that it does not excite unmodeled high frequency dynamics. To put it another way the adaptive loop must remain stable in the presence of <u>unstructured</u> modeling uncertainty which always exists and cannot be adequately modeled in any physical system. On the other hand, the adaptive control system must also be able to provide performance improvement in the case of plant <u>structured</u> uncertainty, typically exhibited when the parameters in the differential equations that are used to model the plant in the low frequency region vary within a bounded set. The adaptive system must exhibit good command-following and disturbance-rejection properties in the low frequency region where the structured model uncertainty predominates.

We believe that there may exist a fundamental conflict in many adaptive control schemes. To compensate for structured uncertainty and performance the adaptive scheme may wish to increase the cross-over frequency. On the other hand, the presence of <u>unstructured</u> uncertainty places an upper bound on the crossover frequency in order to maintain stability. Thus, the

sought- for practical adaptive algorithms must be "smart" enough to recognize this fundamental conflict, and adjust their crossover frequency.

As we have alluded above, the mathematical assumptions that have led to all available adaptive control algorithms have taken into account the existence of structured uncertainty but they have neglected completely the issue of unstructured uncertainty; unhappily, the available algorithms (that we have investigated) are very vulnerable to the presence of unmodeled high-frequency dynamics because the closed loop system because unstable.

If classes of practical adaptive control algorithms were available, then numerous application areas would benefit in both the military and commercial sector. Advances in microprocessor technology allow the engineer to implement <u>in real-time</u> the non-linear, time-varying algorithms necessary to implement the adaptive dynamic compensator necessary to stabilize and improve the performance of a plant with poorly understood characteristics.

### 3.2 Progress to Date [14] to [16]

For the past 2 years an intensive study of characteristics of existing direct adaptive control algorithms has been conducted by Drs. Athans, Stein, Valavani, and Sastry assisted by several students. The initial emphasis was to understand the transient behavior of existing direct adaptive control algorithms and their robustness to unmodeled dynamics and observation noise.

The first phase of this research was devoted to digital simulation studies, which showed that no consistent pattern with respect to the adaptation process could be predicted. Nonetheless the simulation results confirmed our suspicious that the class of adaptive algorithms considered were characterized by

(a) high-frequency control signals characteristic of a high-bandwidth system,

(b) the extreme sensitivity of the algorithms to unmodeled high-frequency dynamics which can results in unstable closed loop behavior

(c) lack of robustness to observation noise.

Motivated by the simulation results a decision was made to initiate an analytical investigation into the nature and properties of several available direct control algorithms. The focus of the analytical effort was to understand

(a) the dependence of the closed loop adaptive system bandwidth upon the amplitude and frequency content of the reference input signal,

(b) the robustness of the adaptive control system to unmodeled high frequency dynamics,

(c) the impact of sensor noise.

To gain basic understanding it was assumed that the controlled plant was a simple first order system; the rationale was that if undesirable performance and robustness characteristics were encountered for first-order

systems one could certainly conjecture that the same problems would arise in more interesting high order systems.

A recent paper [1], describes an analytical technique, based upon linearization, which we call the final approach analysis, that can be used to analyze the dynamic properties of several available direct adaptive control algorithms, both in the continuous-time case and the discrete-time case. In particular this method can be used to predict the behavior of the adaptive systems with respect to parameter convergence, sensitivity to unmodeled dynamics, and impact of observation noise.

As explained in more detail in [1], the final approach analysis method is valid during the final stage of adaptation in which the output error is small. During this phase one can linearize the general nonlinear time-varying differential (or difference) equations linking the dynamics of the output error to those of the parameter adjustment algorithm. One then obtains a set of linear differential or difference equations which are either time-varying or time-invariant depending upon the nature of the reference (command) inputs and outputs. It then becomes possible to analyze the behavior of the linearized dynamics using available results in linear system theory. When the resultant dynamics are time-invariant, even simple root-locus type of plots can be used to predict the asymptotic performance of the adaptive system with respect to oscillatory behavior and possible instability in the presence of unmodeled dynamics.

The final approach analysis has been used to analyze the behavior of the adaptive systems when the algorithms of Narendra and Valavani, Feuer

Feuer and Morse, Narendra, Lin, Valavani, Morse, Narendra, Lin, Landau and Silveira and Goodwin Ramadge, and Caines were employed. For the base line first-order example considered all algorithms considered were found (in different degrees) to suffer from the viewpoint of yielding high-bandwidth closed-loop systems which can excite unmodeled high-frequency dynamics and lead to closed-loop instability.

Since the final approach analysis is based upon dynamic linearization under the assumption that the output error is small, it cannot predict the dynamic behavior of the adaptive system during its transient (start-up) phase. The simulation results suggest that even more complex dynamic effects are present. Thus, we view the final approach analysis as a necessary, but by no means sufficient, step in the analysis and design of adaptive algorithms.

In spite of its limitation, we believe that the final approach analysis is a useful tool since it can predict undesirable characteristics of wide classes of adaptive algorithms in the final phase of the adaption process; these undesirable characteristics are apt to persist (or get even worse) in the transient start-up phase. Moreover, the final approach analysis can suggest ways of modulating the control gains, in a nonlinear manner, to improve performance while retaining the global stability properties of the algorithms in the absence of high frequency modeling errors (structured uncertainty). At this stage of understanding the resultant transient start-up characteristics can be evaluated only by simulation; analytical insights are needed.

### 3.3 Future Directions

Our research has brought to the surface a whole variety of fundamental issues that must be dealt with when one includes high frequency unstructured uncertainty in the adaptive control problems formulation. It is self evident that adaptive control algorithm must be able to control accurately the cross-over frequency of the closed loop system to avoid instability. Also, rapid changes in the parameters associated with the structured uncertainty can result in the instability of the adaptive control loop.

Both frequency-domain and time-domain tools must be used. The ability of adaptive algorithms to tolerate unknown signals in the output variables, that may be due to temporary excitation of high frequency dynamics, must be quantified. It may be necessary to impose constraints on the speed that parameter variations can occur in the structured uncertainty, and to relate the speed of convergence of the adaptive algorithm to that of parameter variations. This in turn would require a comprehensive analysis of the nonlinear time-varying differential or difference equations that are characteristic of popular adaptive control algorithms. To ensure global stability results in the presence of unstructured uncertainty one may have to adapt the conic sector stability results of Safonov to the nonlinear time-varying equations that describe the adaptive control system. In the absence of global stability results, one should develop local ones, which means that one may want to further limit the range and rate of parameter variations in the structured uncertainty. At any rate, the "black box"

approach associated with model reference adaptive control systems, in which no specific assumptions are made upon the nature of the structured uncertainty, will probably be abandoned.

We believe that the explicit inclusion of the unstructured uncertainty into the adaptive control problem will lead to new insights into the desirable and undesirable attributes of existing adaptive control algorithms and, in addition, will point the way to the development of more robust adaptive control algorithms in the presence of unmodeled high-frequency dynamics. More research is also needed to properly account for stochastic additive disturbance as well as for stochastic additive sensor noise.

# 4. THE DEVELOPMENT OF FAILURE DETECTION ALGORITHM DESIGN METHODS

## 4.1 Motivation

As higher and higher performance goals are set for automatic multivariable control systems, there naturally arises a commensurately increased need for high-performance failure detection failure, isolation, and failure identification algorithms. The research described in this section is focused on the major problems related to this need.

As discussed in previous progress reports the focus of our research in failure detection is on bridging the gap between the theoretical methods that have been developed for failure detection and the issues that must be addressed in designing practical failure detection algorithms. In particular our approach has been to formulate and work on analytical problems that directly focus on key issues typically ignored in earlier theoretical studies. The ultimate goals of this research are the development of a deep understanding of practical failure detection and the development of (probably interactive) algorithms for designing failure detection algorithms for large and complex systems.

## 4.2 Progress to Date: [17] to [19]

During this time period we have made significant progress on one of the most critical problems. As mentioned in the previous progress report, Prof. A. Willsky and Mr. Xi-Cheng Lou had initiated a fundamental investigation of the problem of robust failure detection, building upon

the earlier doctoral work of Dr. E.Y. Chow. The results of this research effort have been numerous and significant, and they are described in great detail in the S.M. thesis of Mr. Lou [18] to be completed this month.

The focus of the research of Prof. Willsky, Mr. Lou, and Prof. G.C. Verghese (who has assisted in this effort) has been on the development of methods for generating signals which nominally should be near zero, the basic idea being that a failure detection algorithm can then be based on looking for systematic, nonzero trends in these "parity check" signals. A first question that arises in this context is a deterministic one: if there is no model uncertainty and there is no measurement or process noise, what are the perfect system "parity checks," that is the various linear combinations of lagged sensor outputs and command inputs that are identically zero under normal operating conditions. While several researchers (including Chow and Willsky [17], [19]) had roughly characterized the set of parity checks, no first principles characterization was available which could then be used as the basis for an algorithm which generated these parity checks. Using polynomial descriptions of parity checks, we have now obtained such a characterization (for the case of sensor failures only) and an algorithm which not only generates all possible parity checks for a given system but also does this by finding a complete set of minimal order parity checks (i.e. ones involving the least memory) from which all others can be generated.

While this result is significant, it in fact represents only a comparatively minor aspect in our recent research. In particular, what we have focused our attention on is the generation of "good" parity checks for sensor failure detection when model uncertainties are taken into account. The key point on which our work is based is the observation that in the deterministic case a complete set of all parity checks of order up to some integer p can be specified as the orthogonal projection of the vector of lagged measurements $[y'(k), y'(k+1),...,y'(k+p)]'$ onto a subspace which is orthogonal to the space Z of possible values for this vector under normal operating conditions. Thus finding all parity checks is equivalent to finding the orthogonal complement G of Z. When model parameter uncertainties are present, however, the subspace Z will in general be different for different hypothesized values of the parameters. Conceptually what makes sense in this case is to find a subspace G that is "as orthogonal as possible" to all possible subspaces Z that result from different parameter values. Using the fact that the angle between subspaces can be computed in terms of the maximum singular value of the product G'Z where G and Z are matrices whose columns form orthonormal bases for the subspaces denoted by the same letters, we have been able to formulate a version of the robust parity check problem as the minimization of the maximum singular value of G'Z.

The formulation just described is extremely appealing conceptually and geometrically. It does, however have two drawbacks: (1) the minimax

problem is not easy to solve; and (2) this formulation does not take into account the fact that some directions within the subspace Z may be "preferred" in the sense that the vector of lagged measurements is more likely to lie in particular directions. We have overcome both of these limitations by formulating a variety of other versions of the robust parity check problem. The most important of these uses a priori information (of the unknown-but-bounded variety) in essentially computing the volume of parity check values that would result from a particular choice of G as the unknown parameters are varied over their range of possible values. The resulting algorithm reduces (for a fixed choice of p) to a single singular value decomposition. Furthermore, from this one computation we obtain the optimum choices for G for all possible dimensions. That is, one directly calculates in an ordered fashion the best parity check, the next best parity check which is orthogonal to the first one, the next best which is orthogonal to the first two, etc. The singular vectors in this calculation directly yield the desired parity checks, while the singular values indicate the quality of each parity check. Consequently, what we have uncovered is a quantitative method for measuring the redundancy in an uncertain system.

### 4.3 Future Research Directions

A variety of open problems remain. In particular the work just described has focused completely on the problem of sensor failure detection, for the most part on parameter uncertainties rather than on measurement and driving noise, and essentially entirely on finding parity checks that yield

small values under normal operating conditions rather than on parity checks that yield distinctive trends for particular failures. Recently we have initiated efforts to modify our results to incorporate noise and and to optimize the choice of parity checks which yield small values under normal conditions and which yield large values when particular failures occur. We plan to continue these efforts, and also we plan to extend our results to include actuator failures.

In addition to this line of research, we also intend to resume our efforts in developing methods for computing decision rules for failure detection. Our previous work in this direction, developed by Prof. Willsky and Dr. Chow [17], [19] was limited by the fact that the precise computation of such rules involves the evaluation of numerous multidimensional Gaussian integrals. This is an extremely complex task, and thus there is a need to develop simpler approximations. Recently we have uncovered several promising directions for devising such approximations. The most promising of these involves the use of an approximate model for the evolution of the likelihood ratios for different failure modes. In addition, we also intend to continue our research on approximate performance evaluation for failure detection algorithms using some of our recent results on bounding level crossing probabilities.

# 5. FAULT-TOLERANT OPTIMAL CONTROL THEORY

## 5.1 Motivation

In many present-day control applications there is a strong desire to push performance to the limits imposed by safe operation. In order for such systems to perform successfully, however, it is necessary for them to be fault-tolerant, that is, to be "smart" enough to avoid catastrophes caused by component failures arising from operation at extreme limits or by a failure of the control system to respond correctly when a failure is detected. One way in which to accomplish this is to use a robust control design which sacrifices some performance under normal conditions in order to avoid causing failures or to minimize potential difficulties if a failure should occur. While in some applications this may be acceptable, there are many others in which this sacrifice in performance under normal conditions is unacceptable. What is needed in these problems are control systems which perform at a high level but which are capable of hedging, when necessary, to avoid unsafe conditions and of reorganizing subsequent to a failure in order to minimize the effect of the failure. In our research we are examining two classes of theoretical problems aimed at developing methods for designing fault-tolerant control systems.

## 5.2 Progress to Date [20]

Prof. A. Willsky, Mr. H. Chizeck, and Dr. D. Castanon have been investigating a class of stochastic control problems in which the

probability of failure depends upon the system state and control. The results of this research effort will be reported in the Ph.D. thesis of Mr. Chizeck to be completed in July of 1982.

The basic setting for this research effort is the class of linear systems subject to abrupt changes which are modeled by a finite-state process whose transition probabilities are piecewise-constant functions of the state and control we use quadratic cost criteria which also can depend on the finite-state process. For the most part our detailed research has focussed on scalar problems as this has proven to display a suprisingly rich variety of possible structures. In the last progress report we pointed out the basic structure of the solution to this problem: at any time the optimal cost to go and optimal control are piecewise-quadratic and piecewise-linear, respectively, representing different regimes of operation, some of which correspond to hedging to minimize the effect of failures, others to optimizing performance without regard for possible failures, and still others which represent mixed strategies in which performance is optimized for some period with hedging taking place at a future time.

During the preceding year we have gained a significant amount of additional insight into the problem just described. In particular we have obtained a much more detailed picture of the structure of the optimal solution, we have characterized in detail several qualitatively different types of behavior and have obtained asymptotic results. We have used our

results to obtain computationally efficient algorithms for computing the optimal control policy and high-performance suboptimal policies, and we have in general gained significant understanding into the nature of fault-tolerant control.

Beyond this effort we have also made significant steps in extending our results in two important directions: (1) the inclusion of process noise, and (2) the consideration of higher-dimensional problems. When process noise is included, the piecewise quadratic/linear nature of the solution is lost; however the structure of the solution, combined with the insight we have gained from our earlier work, has allowed us to obtain an accurate suboptimal solution which displays the basic properties of fault-tolerant control in the presence of uncertainties. In the multi-dimensional case, additional complexity arises, since hedging, in the sense of driving the system into a safer region, uses up only one degree of freedom in specifying where the system state is to be driven. Consequently, one must solve constrained optimal control problems in order to obtain optimal hedging control laws. To date we have obtained some qualitative results concerning the structure of the optimal solution and have developed a promising approach for a suboptimal solution to this problem.

In addition to the work described above, Prof. Willsky and Mr. X.C. Lou have been working on another problem of fault-tolerant control. In particular, in designing failure detection systems one typically chooses a decision rule based on tradeoffs among quantities such as false alarm probabilities and expected detection delays. These measures are, however,

only indirect indicators of performance if the detection system is to be used in a control system which is to be reconfigured subsequent to the detection of a failure. In our work we have been considering a problem in which the failure detection decision rule is chosen directly to optimize control system performance. In particular we have considered a problem in which a system, subject to possible failures, is to be controlled by a controller which is constrained in form in that it must choose among several control laws each of which is optimized with respect to a particular mode (normal or failed) of system operation. This problem is a highly nonstandard sequential decision problem, and our results to date indicate that its solution has a variety of interesting aspects, including the fact that the optimal decision rule directly involves the Riccati equation solutions for each of the possible modes of operation.

### 5.3 Future Research Directions

Our present plan is to focus significantly increased attention on the second class of problems just described. In particular, the detailed structure of the optimal decision rule will be investigated as will its dependence on normal and failed system dynamics and performance indices. In addition, simpler, suboptimal control algorithms will be sought. With regard to the first class of problems we described, we have recently formulated a version of the fault-tolerant control problem with discrete process noise. The advantage of this model is that the piecewise-quadratic/linear nature of our earlier problem is maintained. In addition, we plan to continue our efforts in examining the vector version of this problem.

## 6. SPONSOR INTERACTIONS

We strongly believe that frequent interactions between the MIT/LIDS research staff and our NASA sponsors is very important. Long term technical directions that match changing priorities of NASA Centers can be defined; in addition, early dissemination of significant research results is highly desirable. During this reporting period the following visits took place.

Mr. Marc Bodson spent 2 days at the Ames Research Center discussing his research on VTOL control system design in May 1981.

Professor Michael Athans and Dr. Lena Valavani spent 2 days at the Ames Research Center in December 1981. In addition to informal discussions, two seminars were given in the fields of Multivariable Control and Adaptive Control.

Professor Michael Athans visited the Langley Research Center for 2 days in March 1982 where he presented two seminars on Robust Multivariable Control Synthesis and on Progress in Adaptive Control.

Professor Shankar Sastry visited the Ames Research Center in March 1982 where he discussed issues of nonlinear control.

## 7. PUBLICATIONS

The publications listed below are journal and conference papers, reports, and theses published or generated since April 15, 1981 with total or partial support provided by the NASA Ames and Langley Research Centers under NASA grant NGL-22-009-124.

[1] M. Bodson, "Lateral Control System Design for VTOL Landing on a DD963 in High Sea States," S.M. Thesis, MIT, Cambridge, MA, May 1982.

[2] M. Triantafyllou and M. Athans, "Real Time Estimation of the Heaving and Pitching Motions of a Ship Using a Kalman Filter," Proc. OCEANS 81, Boston, Mass., September 1981.

[3] M. Triantafyllou and M. Bodson, "Real Time Prediction of Marine Vessel Motions Using Kalman Filtering Techniques," Proc. Offshore Technology Conference, Houston, Texas, May 1982.

[4] M. Triantafyllou, M. Bodson, and M. Athans, "Real Time Estimation of Ship Motions Using Kalman Filtering Techniques," November 1981, submitted to Journal of Oceanic Engineering.

[5] A.E. Yagle and B.C. Levy, "Multivariable Root Loci on the Real Axis," Report LIDS-P-1112, MIT, July 1981, (submitted to Int. J. of Control).

[6] A.E. Yagle, "Properties of Multivariable Root Loci," LIDS-TH-1090, S.M. Thesis, MIT, June 1981.

[7] A.E. Yagle and B.C. Levy, "Equations for the Angles of Arrival and Departure for Multivariable Root Loci Using Frequency Domain Methods," LIDS-P-1104, June 1981 (submitted to Systems and Control Letters).

[8] S.S. Sastry and C.A. Desoer, "Asymptotic Unbounded Root Loci: Formulae and Computation," (LIDS-P-1137, August 1981) Proc. 20th IEEE Conference on Decision and Control, San Diego, Calif., December 1981.

[9] P.M. Thompson, G. Stein, and A. Laub, "Angles of Multivariable Root Loci," LIDS-P-1147, September 1981 (accepted for publication in IEEE Trans. on Automatic Control).

[10] M.G. Safonov and M. Athans, "A Multiloop Generalization of the Circle Criterion for Stability Margin Analysis," IEEE Trans. on Auto. Control, Vol. AC-26, No. 2, April 1981, pp. 415-422.

[11] N.A. Lehtomaki, "Practical Robustness Measures in Multivariable Control System Analysis," Report LIDS-TH-1093, Ph.D. Thesis, MIT, June 1981.

[12] N.A. Lehtomaki, M. Athans, et.al., "Robustness Tests Utilizing the Structure of Modeling Error," Proc. 20th IEEE Conference on Decision and Control, San Diego, Calif., December 1981.

[13] P. Ng, "On Regulators with Prescribed Degree of Stability," S.M. Thesis, Dept. of Electrical Engineering, M.I.T., Cambridge, MA., August 1981.

[14] C.E. Rohrs, L. Valavani, M. Athans, and G. Stein, "Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms" (LIDS-P-1122), Proc. 20th IEEE Conference on Decision and Control, San Diego, Calif., December 1981.

[15] M. Athans and L. Valavani, "Some Critical Questions about Deterministic and Stochastic Adaptive Control Algorithms," Proc. 6th IFAC Symposium and System Parameter Estimation, Washington, D.C., June 1982.

[16] C.E. Rohrs, Ph.D. Thesis, expected August 1982.

[17] E.Y. Chow and A.S. Willsky, "Sequential Decision Rules for Failure Detection," Rept. No. LIDS-P-1109, Proc. of 1981 Joint Automatic Control Conference, Charlottesville, Va., June 1981; extended version in preparation.

[18] X.C. Lou, "The Failure Projection Method," S.M. thesis, Dept. of Elec. Eng. and Comp. Sci., M.I.T. May 1982.

[19] E.Y. Chow and A.S. Willsky, "Analytical Redundancy and the Design of Robust Failure Detection Systems," submitted to IEEE Trans. on Automatic Control.

[20] H.J. Chizeck, "Fault-Tolerant Optimal Control," Ph.D. thesis, Dept. of Elec. Eng. and Comp. Sci., M.I.T., to be completed July 1982.

[21] P. Moroney, A.S. Willsky, and P.K. Houpt, "Architectural Issues in the Implementation of Digital Compensators," (LIDS-P-1117) Proc. 7th IFAC World Congress, Kyoto, Japan, August 1981, extended version submitted to Automatica.

[22] P. Moroney, A.S. Willsky, and P.K. Houpt, "Roundoff Noise and Scaling in the Digital Implementation of Control Compensators," LIDS-P-1116, August 1981, submitted to Signal Processing.

[23] M.G. Hall, A.V. Oppenheim, and A.S. Willsky, "Time-Varying Parametric Modeling of Speech," submitted to Signal Processing.